

# Robust Face Recognition Measuring 2D Deformations

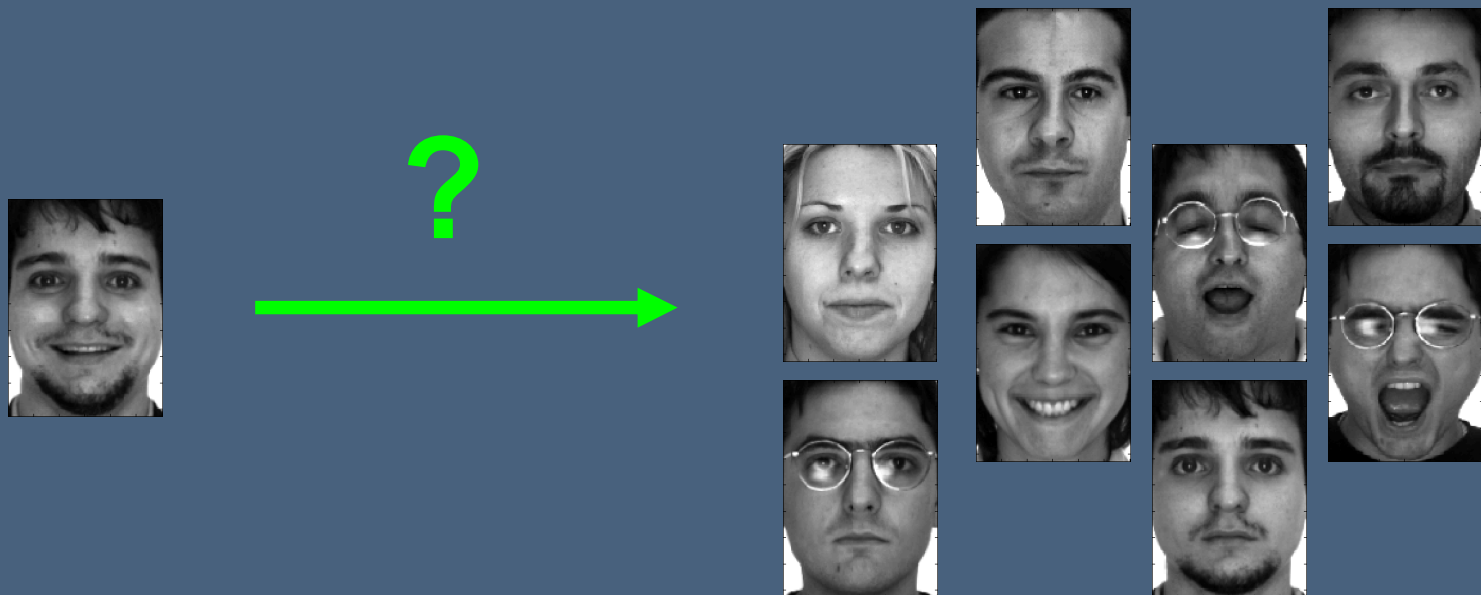
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Applied Mathematics & Statistics, and Scientific  
Computation  
Candidacy Talk  
May 8, 2009

# Outline

- Face Recognition
  - Background
  - Current Deformable Models
    - Feature-based methods
    - Optical Flow
    - Methods using dense correspondences
- A more robust measure of deformation
  - Long range dense correspondences
  - Statistical models of these correspondences and resulting deformations

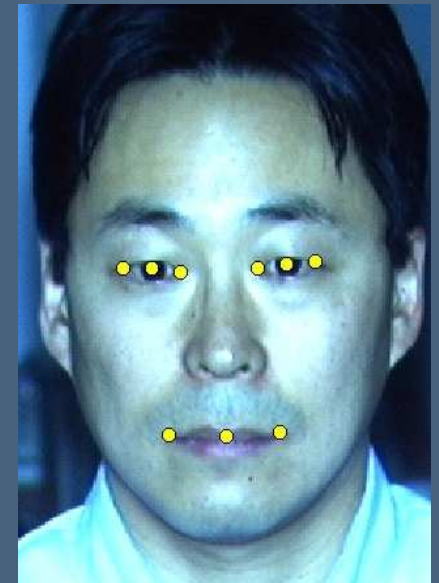
# Problem Statement

- Given a single unknown 2D image of a face, determine the most similar face from a database of known faces.



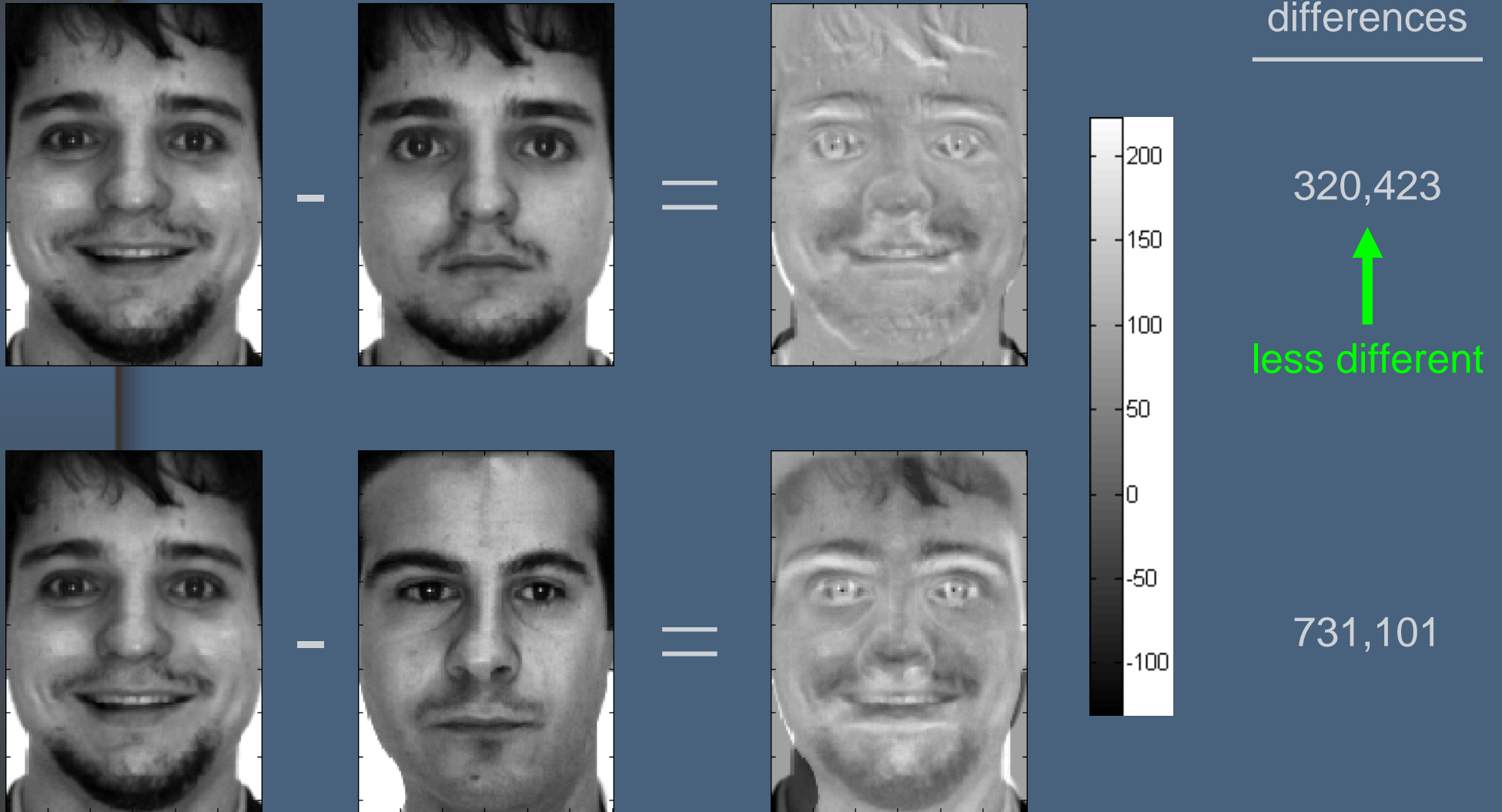
# Background: Image Alignment

- State of the art can reliably detect a small number of corresponding face feature points
- Align images
  - Rotation, translation, scaling
  - Minimize sum of squared distances between individual face point locations and average face point locations
- Image alignment is assumed preprocessing for all methods to follow

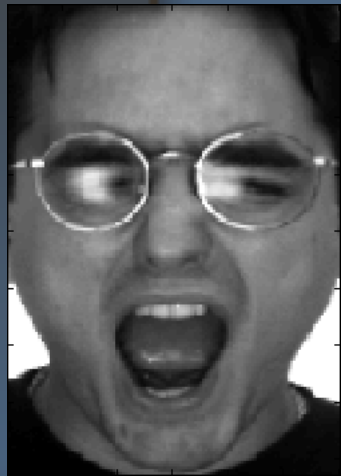


OMRON face points

# Background: Image Differences



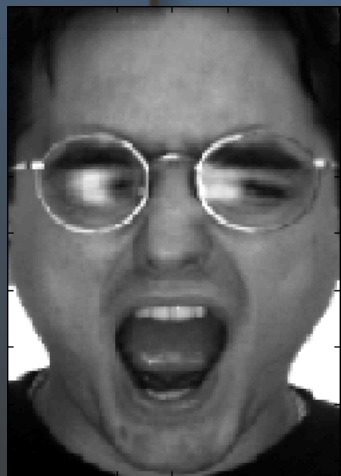
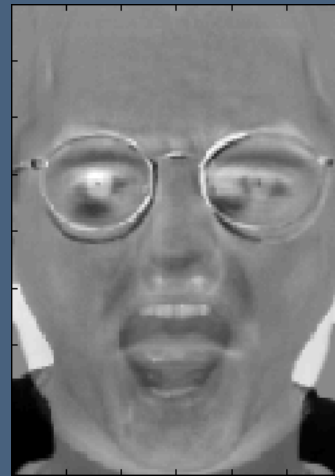
# Background: Image Differences



-



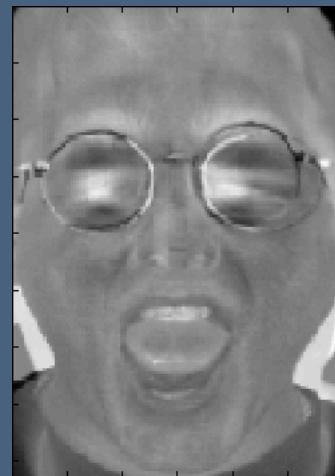
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-

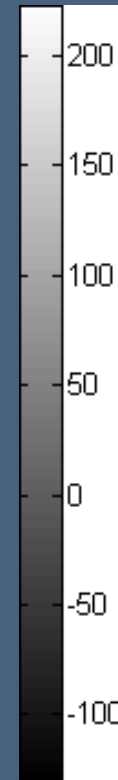


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Sum of pixel differences

---



676,316

less different?



635,435

# Face Representation Algorithms

- First attempts
  - Methods that handle images directly
- Majority of talk
  - Methods that deform input images
  - Measure constructed images and deformations

# Face Representation Algorithms: First Attempts

- Principal Component Analysis (PCA):
  - Find low dimensional linear subspace that captures the most important variations in the dataset

•  $I$ :   $\rightarrow$  

Normalize:  $\text{mean}(I) = 0, \text{var}(I) = 1$

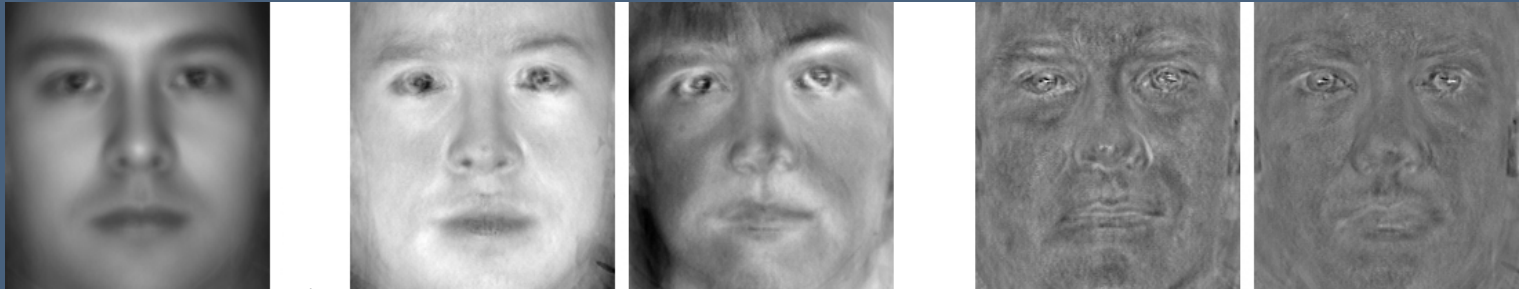
• 
$$\begin{bmatrix} \text{---} \\ \text{---} \\ \vdots \\ \text{---} \end{bmatrix} = USV^T$$

- First  $k$  principal component vectors of  $V$  are the “eigenfaces” of the dataset. Linear combinations provide approximations to true images.



# Face Representation Algorithms: First Attempts

- Some eigenfaces:



average  
face

first two  
eigenfaces

last two  
eigenfaces

- A face projected into its eigenbasis:

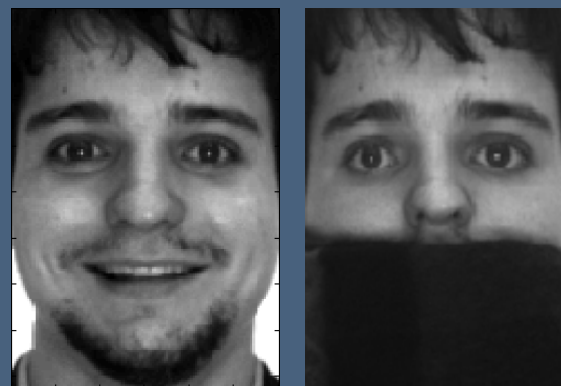


# Face Representation Algorithms: First Attempts

- Linear Discriminant Analysis (LDA):
  - Instead of finding the best subspace representation, find the best classification:
    - Maximize difference between classes  
 $S_B$ : between-class covariance matrix
    - Minimize difference within each class  
 $S_W$ : within-class covariance matrix
    - $I_{LDA} = \omega^T I$ , pick projection  $\omega$  to maximize  $\frac{\omega^T S_B \omega}{\omega^T S_W \omega}$

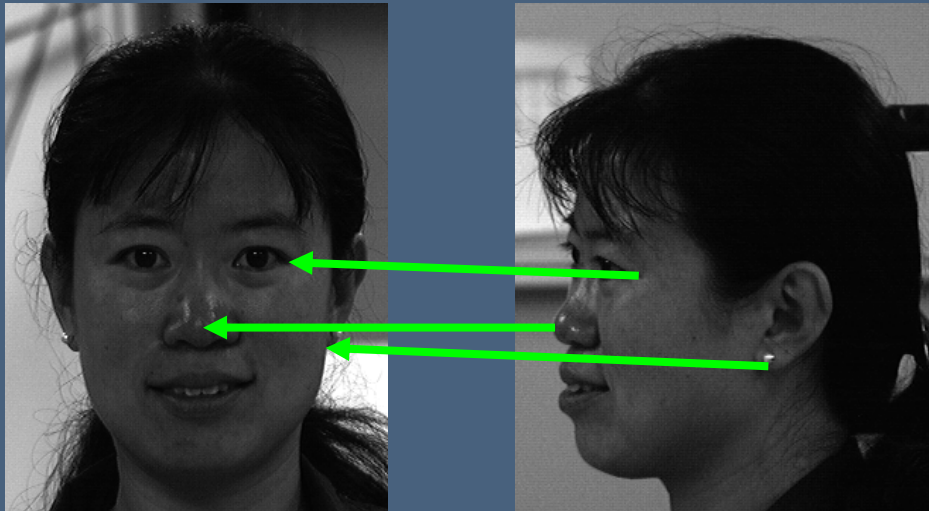
# Problems with Pixels

- Pixel-based methods fail when variations in *pose*, *expression*, *lighting* and *occlusions* are introduced.



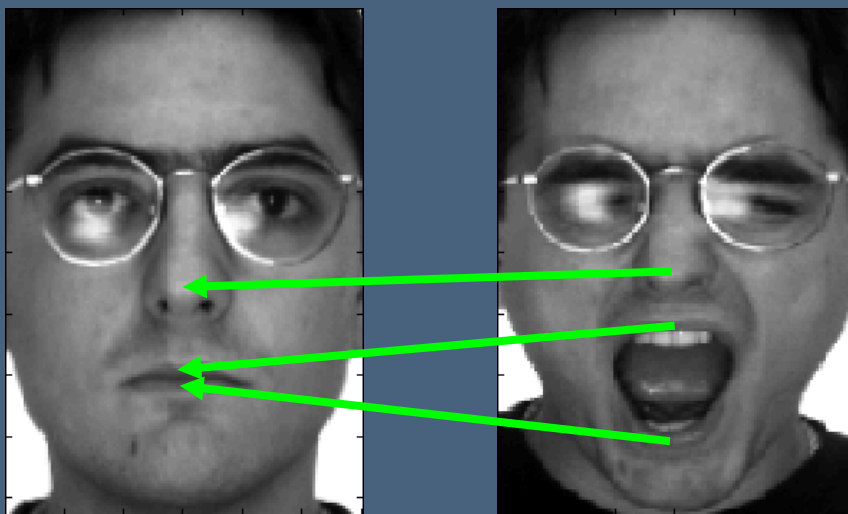
- Want to warp input face to standard expression and pose before calculating the image difference.

# Finding Correspondences



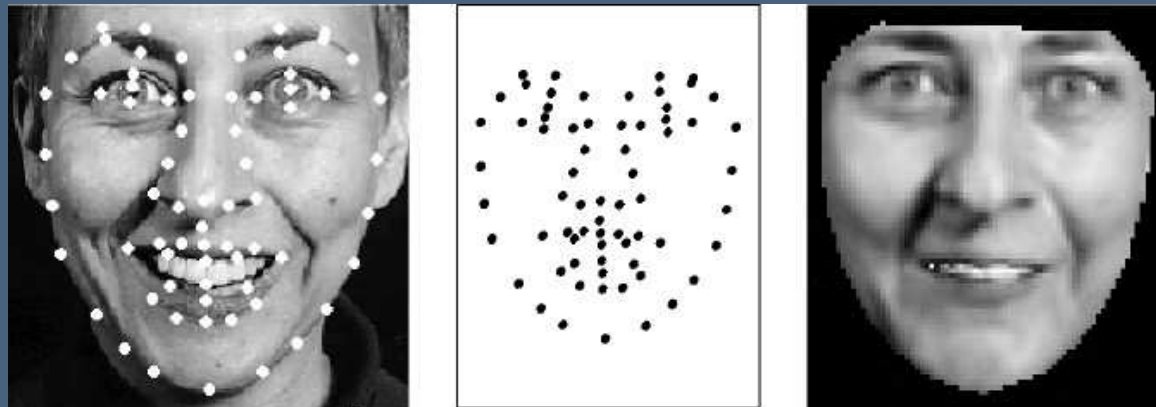
How to determine correspondences?

What to do with them once they are found?



# Active Appearance Models

- Separate Shape (location) information from Texture (intensity) information:
  - Identify corresponding feature points in each image
  - Warp points to average locations, interpolate all other points
  - Map texture values respectively for “shape-free patch”



original labeled  
image

average point  
locations

shape-free  
image

# Active Appearance Models

- An individual image has shape vector  $x$  and texture vector  $t$ , where:

$$x = \bar{x} + Q_s c$$

$$t = \bar{t} + Q_t c$$

$Q_s$ : modes of shape variation

(PCA over point locations)

$Q_t$ : modes of texture variation

(PCA over warped images)


$c$ : image-specific parameter values

# Active Appearance Models

- Iterate model to generate good match to input image
  - Residual error at iteration  $m$ :  $r(c_m) = t_0 - t_m$ 
    - $t_0$  = input image texture
    - $t_m$  = current warped model texture
  - Update parameters:  $c_{m+1} = c_m + \delta c_m$

where  $\delta c_m$  is chosen to minimize  $\|r(c_m + \delta c_m)\|^2$   
using the first order Taylor expansion:

$$r(c_m + \delta c_m) = r(c_m) + \frac{\partial r}{\partial c} \delta c$$

 estimated from training  
data

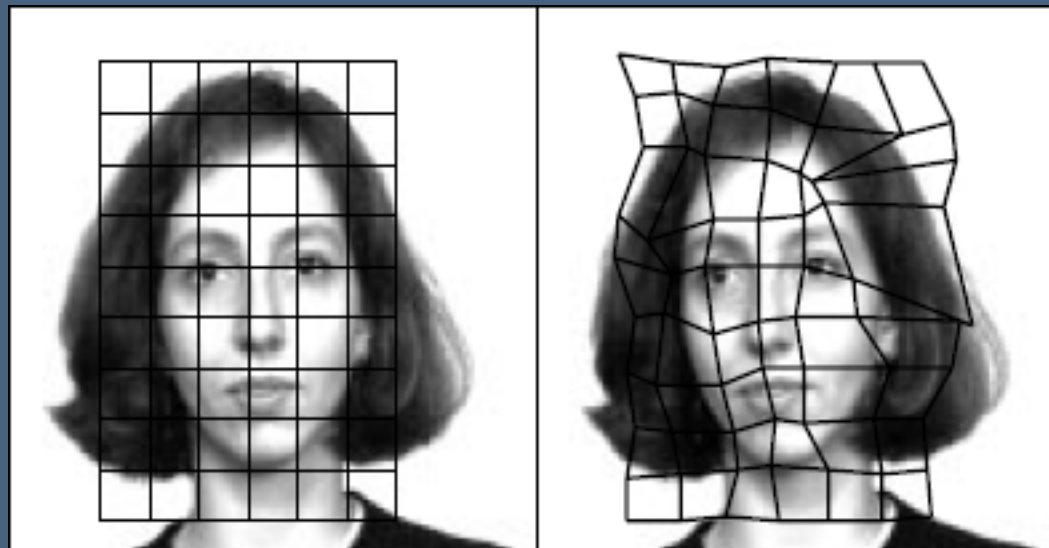
# Automatic Correspondences

- Unreasonable to expect large number of feature point correspondences
- State of the art can reliably detect a small number of face feature points
  - Useful for image alignment
  - Insufficient for warping
- Would like to automatically obtain correspondences



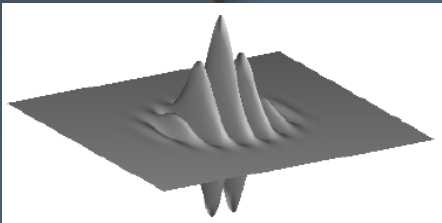
# Dynamic Link Matching

- To find correspondences for comparison
- Fit a uniform grid of nodes over a face, adjusting each node locally to best fit a model.



# Dynamic Link Matching

- Each node = “jet”, a vector:
  - Gabor wavelet convolution with the image
  - Gabor wavelets are a “good approximation to the sensitivity profiles of neurons found in the visual cortex” of the brain
  - 5 scales
- Fit new image jet  $J^I$  with model jet  $J^M$ :



$$\max C_v(J^I, J^M) = \frac{\langle J^I, J^M \rangle}{\|J^I\| \|J^M\|}$$

# Dynamic Link Matching

- Also want to minimize the image distortion
  - Distance between nodes:

$$\Delta_{ij} = \vec{x}_j - \vec{x}_i$$

- Overall distortion:

$$\min C_e(\Delta_{ij}^I, \Delta_{ij}^M) = (\Delta_{ij}^I - \Delta_{ij}^M)^2$$

# Dynamic Link Matching

- Total cost to be minimized:

$$C(x_i^I) = \lambda \sum_{(i,j) \in E} C_e(\Delta_{ij}^I, \Delta_{ij}^M) - \sum_{i \in V} C_v(J^I(x_i^I), J_i^M)$$

distortion penalty constant      minimize distortions      maximize node match similarity

- Optimize via simulated annealing
  - Randomly shift the nodes

# Pictorial Structures

- Learn cost function for deformations specific to faces, depends on:
  - Local image similarity
  - Amount of deformation required to arrive at this similarity
- Consider connections between few higher level “parts”
  - Unlike other algorithms, this method is only for face *detection*

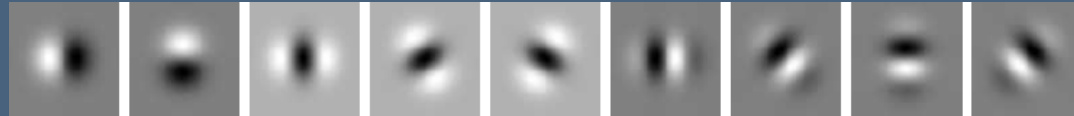


# Pictorial Structures

- “Part”

- 27-D vector

- Gaussian derivative filters



- Varies order, orientation and scale

- Learn what parts look like from labeled training examples

# Pictorial Structures

- Best match of new image to model:

$$L^* = \operatorname{argmin}_L \left( \sum_{i \in V} m_i(\ell_i) + \sum_{(i,j) \in E} d_{ij}(\ell_i, \ell_j) \right)$$

mismatch to model when  
part  $v_i$  is placed at location  $\ell_i$

deformation of the model  
between parts  $v_i$  and  $v_j$   
(Mahalanobis correlation  
distance)

- Method detects faces
  - Not discriminative enough for identification

# Dense Correspondences

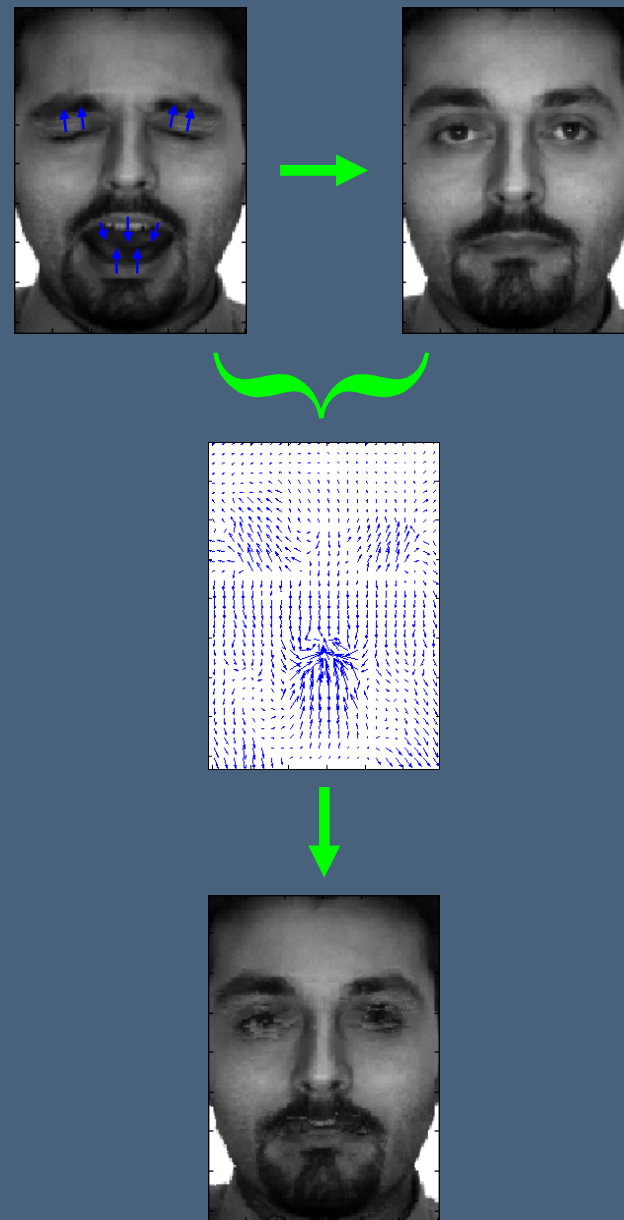
- Match every point in new face to some point in known face





# Dense Correspondences

- Match every point in new face to some point in known face.
- Optical flow
  - Determine the displacement of every pixel in the first image to the most similar pixel in the second
  - Return  $[u, v]$  vector for each point
    - Vector field over the image
  - Assume images are similar
  - Assume intensity is preserved between corresponding patches



# Optical Flow

- Intensity constraint equation:

$$I(x + \delta x, y + \delta y, t + \delta t) = I(x, y, t)$$

- Taylor series:

$$I(x + \delta x, y + \delta y, t + \delta t) = I(x, y, t) + \frac{\partial I}{\partial x} \delta x + \frac{\partial I}{\partial y} \delta y + \frac{\partial I}{\partial t} \delta t$$

$$0 = \frac{\partial I}{\partial x} u + \frac{\partial I}{\partial y} v + \frac{\partial I}{\partial t}$$

Optical flow values to be returned

$\frac{\partial I}{\partial x}, \frac{\partial I}{\partial y}, \frac{\partial I}{\partial t}$  calculated using finite differences of pixels

- Optical Flow equation:  $\nabla I \cdot \vec{v} + I_t = 0$

$$\text{Let } E_b = \nabla I \cdot \vec{v} + I_t$$

- Need second constraint to explicitly solve for  $u, v$

# Optical Flow

- Horn and Schunk

- Enforce smoothness by minimizing gradient of flow:

$$E_c^2 = \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial u}{\partial y} \right)^2 + \left( \frac{\partial v}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial y} \right)^2$$

- To solve:

$$\min \int \int (E_b^2 + \lambda E_c^2) dx dy$$

# Optical Flow

- Problems at motion boundaries



First two frames in video sequence




Least squares estimate of horizontal flow  
(Horn and Schunk)



Robust gradient estimate of horizontal flow  
(Black and Anandan)

# Optical Flow

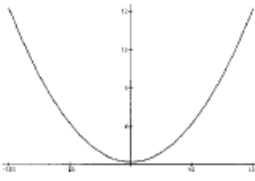
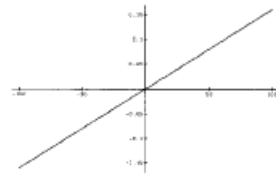
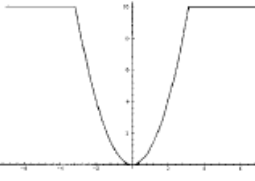
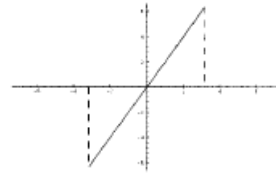
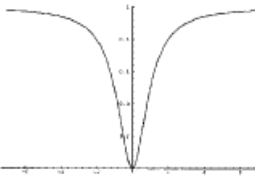
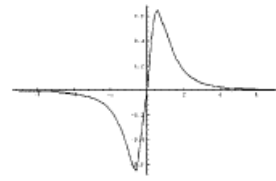
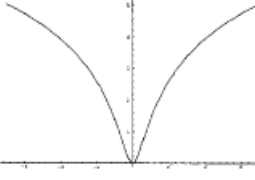
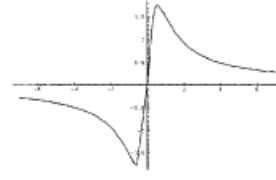
- Black and Anandan
  - Robust Statistics
    - Exclude outliers to handle object boundaries
    - Incorporate robust  $\rho$ -function (error) and its derivative  $\psi$  (proportional to the influence function)

$$\min \int \int (\rho_b(E_b^2) + \lambda \rho_c(E_c^2)) dx dy$$


function to limit influence of outliers

# Optical Flow

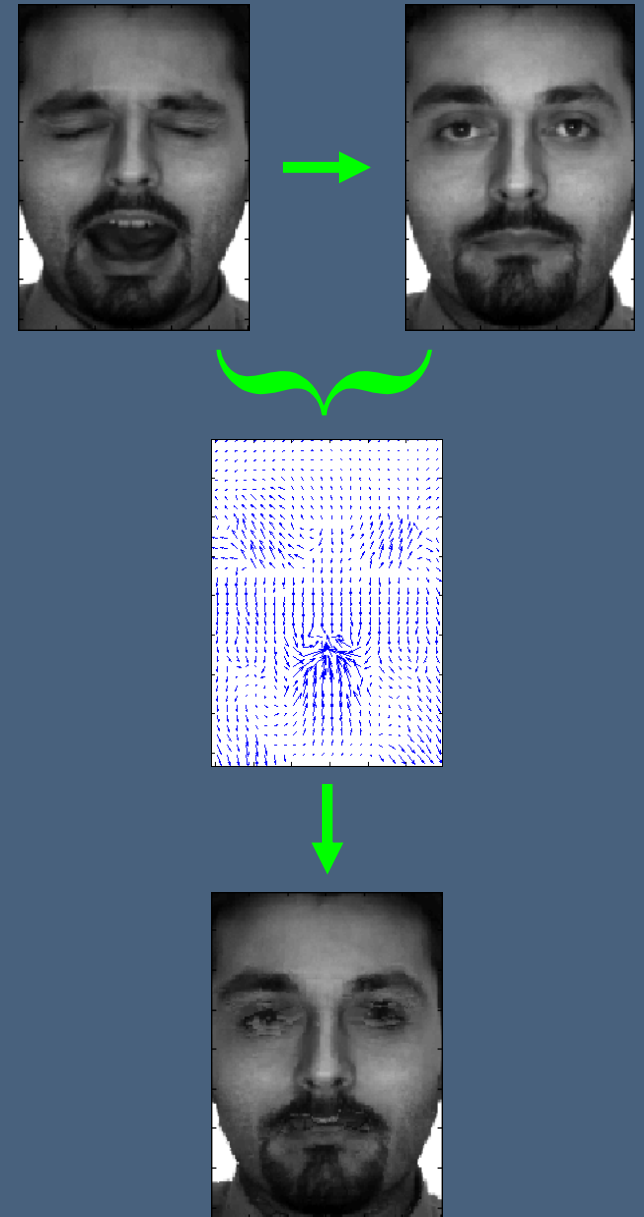
$\rho$ -function  
(error)

<i>(a). Quadratic (L2 Norm):</i>	
$\rho(x) = x^2$	$\psi(x) = 2x$
	
<i>(b). Truncated Quadratic (skipped mean) [12]:</i>	
$\rho(x, \alpha, \lambda) = \begin{cases} \lambda x^2 & \text{if }  x  < \frac{\sqrt{\alpha}}{\sqrt{\lambda}}, \\ \alpha & \text{otherwise.} \end{cases}$	$\psi(x, \alpha, \lambda) = \begin{cases} 2\lambda x & \text{if }  x  < \frac{\sqrt{\alpha}}{\sqrt{\lambda}}, \\ 0 & \text{otherwise.} \end{cases}$
	
<i>(c). Geman &amp; McClure [20]:</i>	
$\rho(x, \sigma) = \frac{x^2}{\sigma + x^2}$	$\psi(x, \sigma) = \frac{2x\sigma}{(\sigma + x^2)^2}$
	
<i>(d). Lorentzian:</i>	
$\rho(x, \sigma) = \log \left( 1 + \frac{1}{2} \left( \frac{x}{\sigma} \right)^2 \right)$	$\psi(x, \sigma) = \frac{2x}{2\sigma^2 + x^2}$
	

$\psi$ -function =  
derivative of  $\rho$   
(proportional to  
the influence  
function)

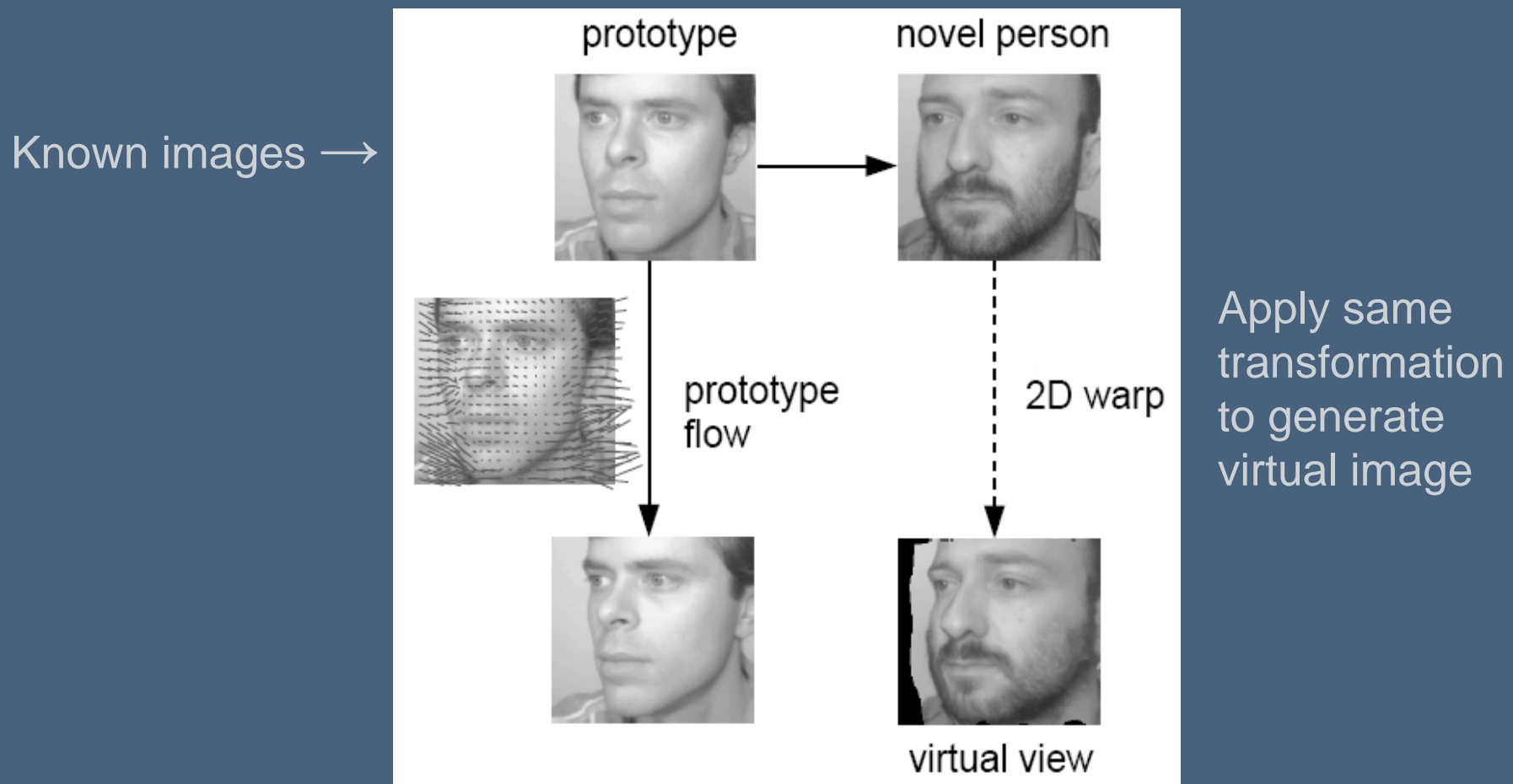
# Methods Using Dense Correspondences

- Use optical flow to obtain corresponding pixel for every point in an image.



# Warping A Single Image

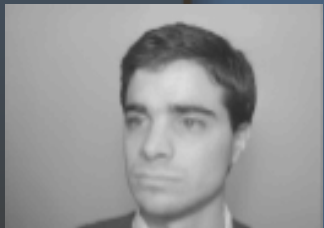
- Use prior knowledge of face pose change to warp a single known image to a new artificial image.





# Warping A Single Image

- Algorithm to build database:
  - Have single image of most people
  - Find correspondence between new face and known face
    - Provide key features by hand, interpolate for other points
    - Similar to Active Appearance Models
  - Apply known transformations to generate many virtual views
    - Optical Flow at each point

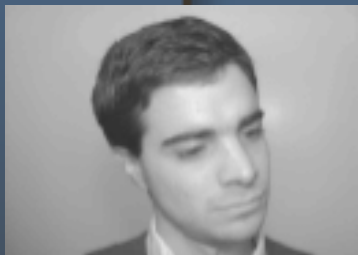


→  
Optical  
Flow



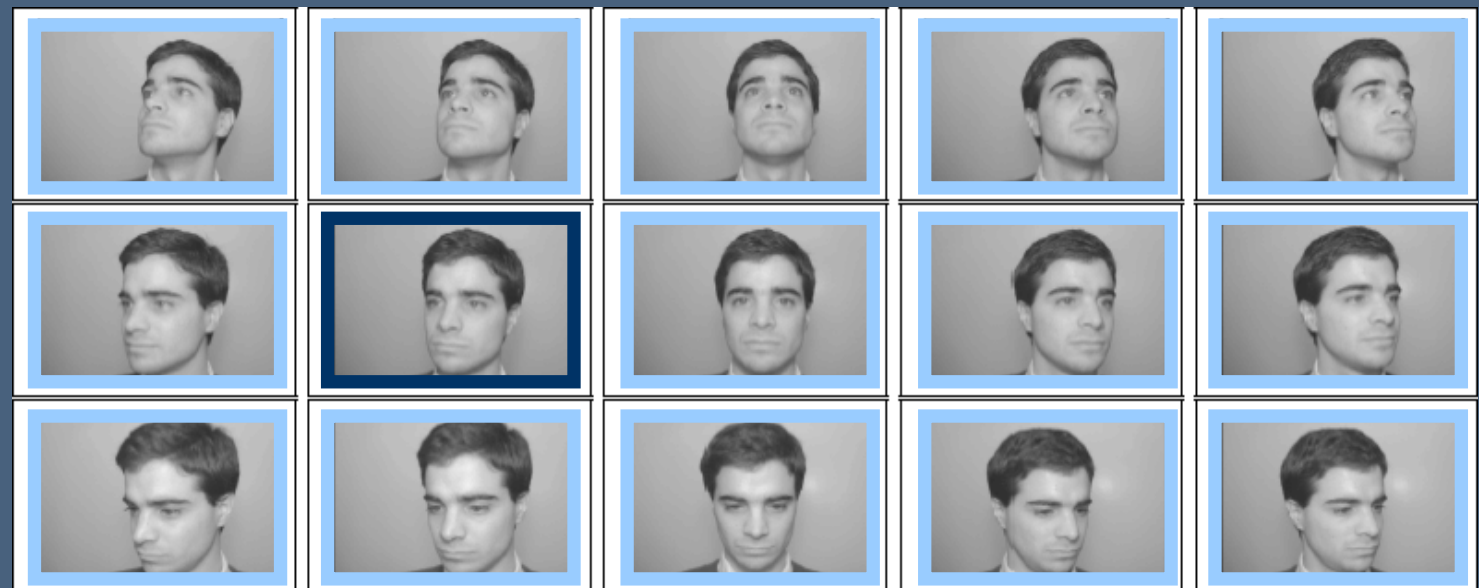
# Warping A Single Image

- Testing:
  - Compare new image to most similar pose of every individual in database
  - Nearest neighbor wins



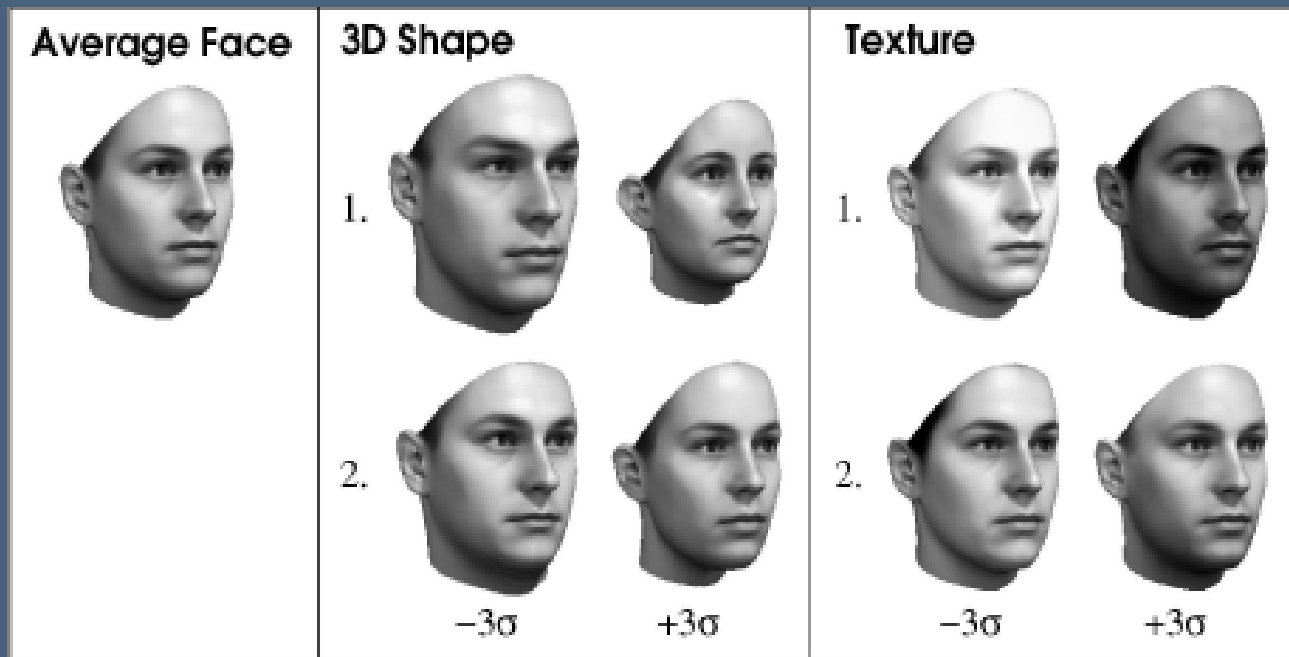
*known  
image*

*virtual  
image*



# 3D Morphable Model

- A “state of the Art” method solving the correspondence problem under pose and lighting variation
- Statistical 3D model instead of several 2D images



adjusting 1<sup>st</sup> principal component

adjusting 2<sup>nd</sup> principal component

PCA on 3D vector describing how a specific point differs from model average of that point

PCA on intensity value at each point

# 3D Morphable Model

- $m$  significant eigenvectors define variation of shape  $S$  and texture  $T$
- Influence of each dimension on a particular face defined by coefficient vectors  $\alpha$  and  $\beta$

$$s = \bar{s} + \sum_{i=1}^{m-1} \alpha_i S_i$$

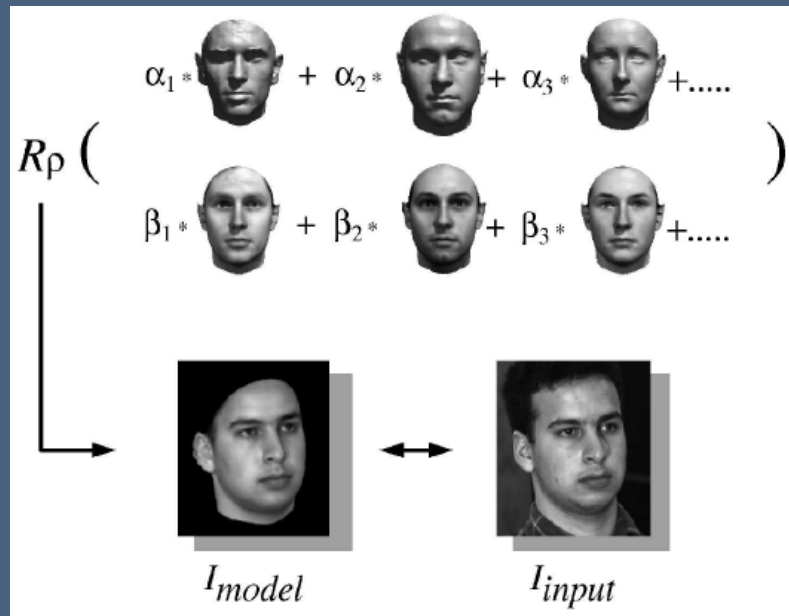
$$t = \bar{t} + \sum_{i=1}^{m-1} \beta_i T_i$$

- Construct synthetic image to closely match unknown face image
  - Minimize sum of squared distances between real and synthetic pixel intensities

# 3D Morphable Model

- Construct model to match image:
  - a posteriori estimate via Bayes:

$$\max P(\alpha, \beta, \rho | I_{in}, F) \sim \max P(I_{in}, F | \alpha, \beta, \rho) \cdot P(\alpha, \beta, \rho)$$

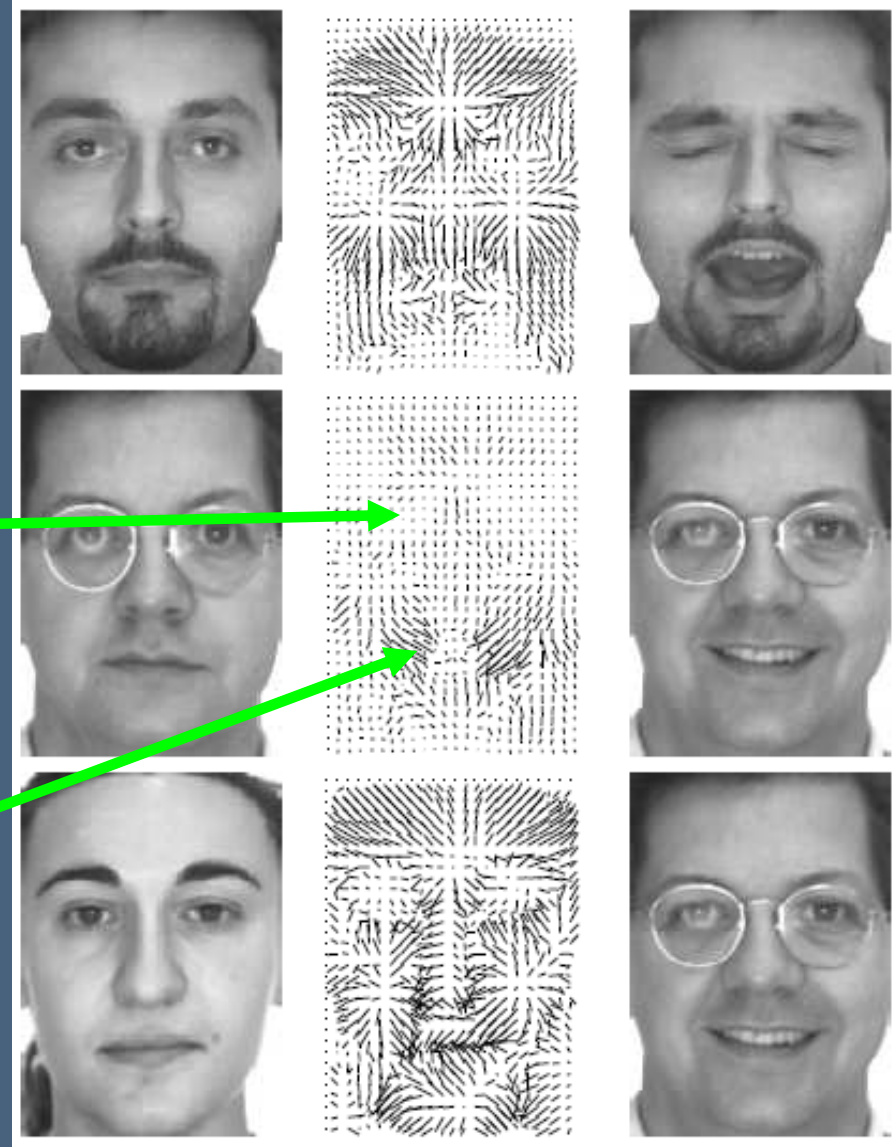


$\alpha$  = shape control parameters  
 $\beta$  = texture control parameters  
 $\rho$  = pose control parameters  
 $I_{in}$  = new image  
 $F$  = small set of feature points  
found during preprocessing

- Match constructed model to known person:
  - Compare model coefficients

# Examine the Optical Flow

- Martinez: Weight importance of pixels by how much they deform
  - Small change: important for recognition
  - Large change: ignore



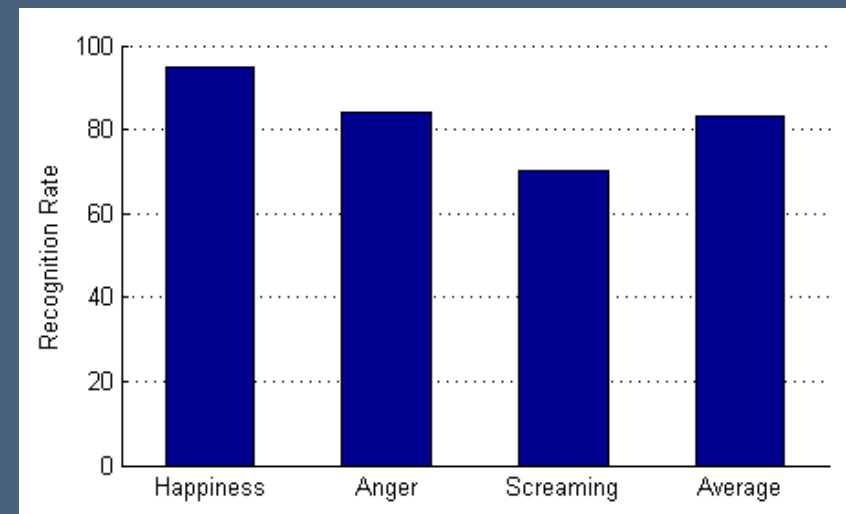
# Examine the Optical Flow

- Weighting scheme
  - Compare new image  $T$  to known images  $I_n$

$$F_n = \text{OpticalFlow}(I_n, T)$$

$$W_{n,i} = \max_i \|F_{n,i}\| - \|F_{n,i}\| \quad (\text{weight for each pixel } i)$$

$$C_n = \|W_n(I_n - T)\| \quad (\text{cost to match } T \text{ to } I_n)$$



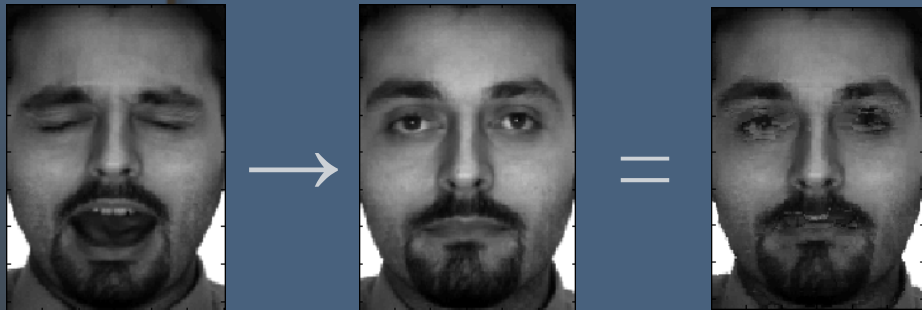
# Limitations of Current Approaches

- Methods using dense correspondences only measure resulting image similarity
- Optical flow meant to solve the small motion correspondence problem
  - No reason to expect it to work for large pose or expression changes
- Need statistical models of deformation change due to expression/pose of same person vs change in identity

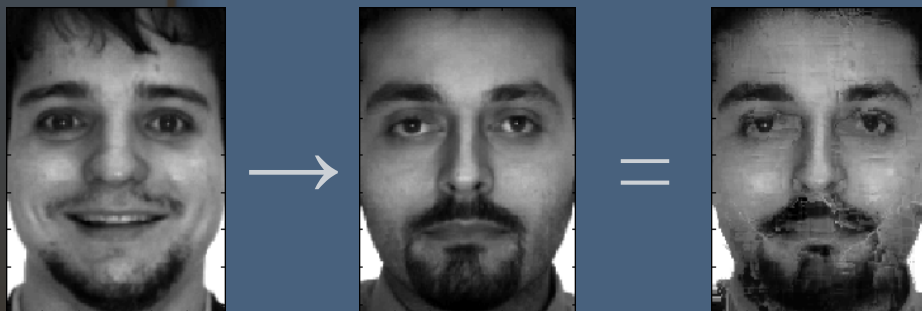


# Measuring the Deformation

- A successful face recognition system should consider:
  - Similarity between images
  - Amount and type of deformation required to achieve this similarity



smaller  $\sum \| \text{Optical Flow} \|_2$



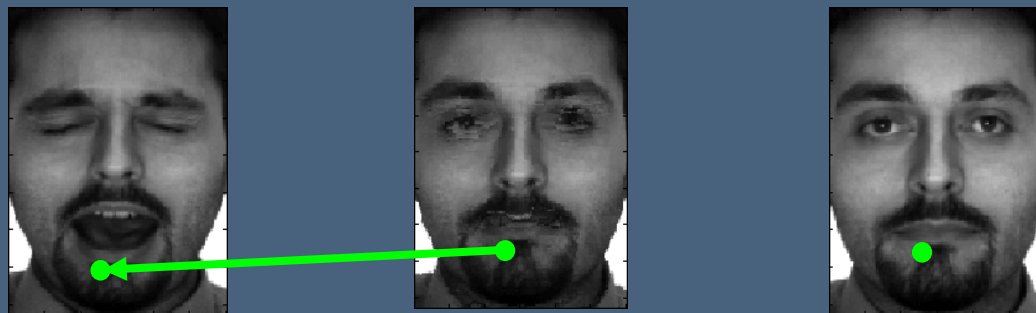
smaller  $\sum \| I_1 - I_2 \|_2$

# Measuring the Deformation

- Similarity between images  $I$  and  $J$ 
  - Let  $v$  be a transformation defined on every pixel of  $I$  such that  $v(I) \approx J$



- For each pixel  $x$  in  $J$ , the corresponding pixel in  $I$  is  $I(v^{-1}(x))$



# Measuring the Deformation

- Similarity between images  $I$  and  $J$ , for all points  $x$ :

$$d(I(x), J(x)) = \|J(x) - I(v^{-1}(x))\|_2 + \lambda \|v(x)\|_g$$



deformed image intensity difference



measure of deformation

- To define:
  - Deformation  $v$
  - Deformation norm  $g$
  - Relative weighting  $\lambda$

# Measuring the Deformation

- Deformation  $v$ :
  - Traditional optical flow
  - Longer range dense correspondence
- Deformation norm  $g$ :
  - Optical flow: any metric defined on a vector field,  $\sum_i \|v_i\|_2, \dots$
  - New field?
- Relative weighting  $\lambda$ :
  - Implicit using Machine Learning techniques
  - Learn from training set
  - Incorporate into  $g$

# Measuring the Deformation

- Previous methods
  - Dynamic Link Matching

$$C(x_i^I) = \lambda \sum_{(i,j) \in E} C_e(\Delta_{ij}^I, \Delta_{ij}^M) - \sum_{i \in V} C_v(J^I(x_i^I), J_i^M)$$

↑  
minimize distortions

↑  
maximize node match similarity

- Pictorial Structures

$$L^* = \operatorname{argmin}_L \left( \sum_{i \in V} m_i(\ell_i) + \sum_{(i,j) \in E} d_{ij}(\ell_i, \ell_j) \right)$$

↑  
part-to-model mismatch

↑  
model deformation

# Optical Flow Limitations

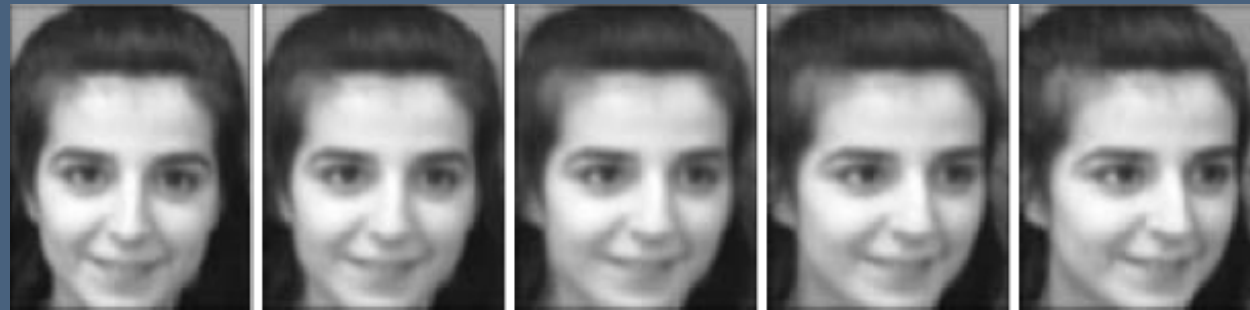
- Optical flow meant to solve small motion correspondence problem
  - Correspondence between faces involves different set of requirements
- Alternative method meant to handle larger changes:
  - Deformations through Lie group action

# Deformations Through Lie Group Action

- Image: continuous Riemannian manifold
- Lie group: diffeomorphisms of the manifold
  - The possible image deformations
- Lie algebra: vector space of infinitesimal steps in the direction of these deformations
  - Continuous vector fields deforming the image
- Geodesic: the deformation requiring the least energy ( $v$ )

# Deformations Through Lie Group Action

- Energy  $E = \min_v \left( \int_0^1 \left\| \frac{\partial I}{\partial t} \right\|_2^2 dt + \int_0^1 \|v_t\|_g^2 dt \right)$
- A geodesic obtained by minimizing the energy between two given images:



input 1  
 $t = 0$

input 2  
 $t = 1$

path generated by algorithm



# Future Research

- Define robust long range dense correspondences between images.
- Build statistical models of these correspondences and resulting deformations.
- Solve image classification problems using this information.